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Symmetrically-Mixed Analysis of Electrical Networks with Switching Elements

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Abstract

The symmetrically-mixed form of Ohm's law including switching elements and the symmetrically-mixed analysis thereon are introduced. And further the practical examples on resistance networks with time-variable switching elements are described.

1. Notations[1) 2)]

Notations in this paper are based on tensor geometry, but the reader may regard them as a mere matrix expression.

(i) Names of graph elements are as follows:

branches : 1, 2, ……, n as fixed indices;
κ, λ, μ, ν …… as free indices
(and so on)

independent points : $\dot{1}$, $\dot{2}$, ……, $\dot{m}$;
a, b, ……

independent loops : $\underset{\cdot}{1}$, $\underset{\cdot}{2}$, ……, $\underset{\cdot}{k}$;
p, q, ……

(ii) The *connection matrix* is denoted by $\mathbf{D}$, D^{a}_{κ}, and the *loop matrix* by $\mathbf{R}$, R^{λ}_{q}.

(iii) The matrix expressions of the electrical quantities are as follows:

$$\begin{cases} \mathbf{i},\ i^{\kappa} : \text{branch currents} \\ \mathbf{u},\ u_{\lambda} : \text{branch voltoge-drops} \end{cases}$$

$$\begin{cases} \mathbf{s},\ s^{\kappa} : \text{branch-current sources} \\ \mathbf{e},\ e_{\lambda} \quad \text{branch-voltage sources} \end{cases}$$

$$\begin{cases} \mathbf{S}, S^{\alpha} = D^{\alpha}_{\kappa}\, s^{\kappa} & : \textit{point-current sources} \\ \mathbf{E}, e_{q} = R^{\lambda}_{q}\, e_{\lambda} & : \textit{loop-voltage sourees} \end{cases}$$

(iv) The fundamental equations of an electrical network are written as follows :

$$\begin{array}{llll} \textit{Current law} & : \mathbf{D}\ \mathbf{i} = \mathbf{S} & : & D^{\alpha}_{\kappa}\, i^{\kappa} = s^{\alpha} \\ \textit{Voltage law} & : \mathbf{R}\ \mathbf{u} = \mathbf{E} & : & R^{\lambda}_{q}\, u_{\lambda} = e_{q} \\ \textit{Ohm's law} & : \mathbf{i} = \mathbf{y}\ \mathbf{u} & : & i^{\kappa} = y^{\kappa\lambda}\, u_{\lambda} \\ & \text{or } \mathbf{u} = \mathbf{z}\ \mathbf{i} & : & u_{\lambda} = z_{\lambda\kappa}\, i^{\kappa} \end{array}$$

In these equations, we adopt the summation convention. If an index appears twice in the same term once as a subscript and once as a superscript, the summation sign $\sum$ will be ommitted.

2. Ohm's law[3)]

The *original form of Ohm's law* is written as

$$\mathbf{u} = \mathbf{z}\ \mathbf{i}\ :\ u_{\lambda} = z_{\lambda\kappa}\, i^{\kappa} \tag{1}$$

$$\text{or } \mathbf{i} = \mathbf{y}\ \mathbf{u}\ :\ i^{\kappa} = y^{\kappa\lambda}\, u_{\lambda} \tag{2}$$

where u_{λ} is the *branch voltage-drop* of branch λ, i^{κ} the *branch current* of branch κ, $z_{\lambda\kappa}$ the *branch impedance*, and $y^{\kappa\lambda}$ the *branch admittance*.

For example, in Fig. 1, Eq. (1), (2) is written as follows:

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} = \begin{bmatrix} z_1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & z_2 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & z_3 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & z_4 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & z_5 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & z_6 \end{bmatrix} \begin{bmatrix} i^1 \\ i^2 \\ i^3 \\ i^4 \\ i^5 \\ i^6 \end{bmatrix}$$

or

$$\begin{bmatrix} i^1 \\ i^2 \\ i^3 \\ i^4 \\ i^5 \\ i^6 \end{bmatrix} = \begin{bmatrix} y^1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & y^2 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & y^3 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & y^4 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & y^5 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & y^6 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix}$$

As a matter of course, it holds that

$$\mathbf{y}\ \mathbf{z} = \mathbf{1} \ \text{ and } \mathbf{z}\ \mathbf{y} = \mathbf{1} \tag{3}$$

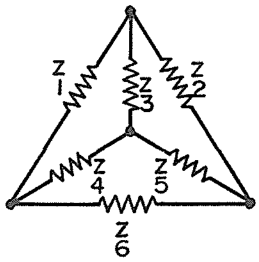

Fig. 1

where **1** is the unit matrix.
Namely, it holds that

$$\begin{bmatrix} z_1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & z_2 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & z_3 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & z_4 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & z_5 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & z_6 \end{bmatrix} \begin{bmatrix} y^1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & y^2 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & y^3 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & y^4 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & y^5 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & y^6 \end{bmatrix} = \begin{bmatrix} 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 \end{bmatrix}.$$

3. Mixed expression of Ohm's law[4)]

In Fig. 1, if the Ohm's law is written by

$$\begin{bmatrix} i^1 \\ i^2 \\ i^3 \end{bmatrix} = \begin{bmatrix} y^1 & \cdot & \cdot \\ \cdot & y^2 & \cdot \\ \cdot & \cdot & y^3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad : \quad i^{\kappa'} = y^{\kappa'\lambda'} u_{\lambda'}$$

and

$$\begin{bmatrix} u_4 \\ u_5 \\ u_6 \end{bmatrix} = \begin{bmatrix} z_4 & \cdot & \cdot \\ \cdot & z_5 & \cdot \\ \cdot & \cdot & z_6 \end{bmatrix} \begin{bmatrix} i^4 \\ i^5 \\ i^6 \end{bmatrix} \quad : \quad u_{\lambda''} = z_{\lambda''\kappa''} i^{\kappa''}$$

we have

$$\left[\begin{array}{ccc|ccc} \overset{1}{y} & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \overset{2}{y} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \overset{3}{y} & \cdot & \cdot & \cdot \\ \hline \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 \end{array}\right] \left[\begin{array}{c} u_1 \\ u_2 \\ u_3 \\ \hline u_4 \\ u_5 \\ u_6 \end{array}\right] - \left[\begin{array}{ccc|ccc} 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ \hline \cdot & \cdot & \cdot & \underset{4}{z} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \underset{5}{z} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \underset{6}{z} \end{array}\right] \left[\begin{array}{c} i^1 \\ i^2 \\ i^3 \\ \hline i^4 \\ i^5 \\ i^6 \end{array}\right] = \left[\begin{array}{c} \cdot \\ \cdot \\ \cdot \\ \hline \cdot \\ \cdot \\ \cdot \end{array}\right].$$

The above relation is denotad by

$$\mathbf{p}\,\mathbf{u} - \mathbf{q}\,\mathbf{i} = \mathbf{0} \;:\; p^{\mu\lambda}\, u_\lambda - q^{\mu}{}_{\kappa}\, i^\kappa = 0 \tag{4}$$

and

$$p^{\mu\lambda} = \left[\begin{array}{ccc|ccc} \overset{1}{y} & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \overset{2}{y} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \overset{3}{y} & \cdot & \cdot & \cdot \\ \hline \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 \end{array}\right], \quad q^{\mu}{}_{\kappa} = \left[\begin{array}{ccc|ccc} 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ \hline \cdot & \cdot & \cdot & \underset{4}{z} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \underset{5}{z} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \underset{6}{z} \end{array}\right].$$

Generally, the above $p^{\mu\lambda}$ and $q^{\mu}{}_{\kappa}$ are written as follows:

$$p^{\mu\lambda} = \left[\begin{array}{c|c} y^{\kappa'\lambda'} & 0 \\ \hline 0 & 1 \end{array}\right], \quad q^{\mu}{}_{\kappa} = \left[\begin{array}{c|c} 1 & 0 \\ \hline 0 & z_{\lambda''\kappa''} \end{array}\right] \tag{5}$$

where symbol 1 is the unit matrix, and 0 the zero matrix. Eq. (4) is called the "*mixed expression of Ohm's law*" and the graphical determination has been given by the authors[4].

4. Symmetricaiiy-mixed expression of Ohm's law[5]

Provided that

$$p^{\mu\lambda} \;:\; \mathbf{p} = (\mathbf{z}+\mathbf{1})^{-1} \tag{6}$$

and $$q^{\mu}{}_{\kappa} \;:\; \mathbf{q} = (\mathbf{y}+\mathbf{1})^{-1} \tag{7}$$

the Ohm's law is expressed as follows:

$$\mathbf{p}\ \mathbf{u}-\mathbf{q}\ \mathbf{i} = \mathbf{0}\ :\qquad p^{\mu\lambda}\ u_{\lambda} - q^{\mu}{}_{\kappa}\ i^{\kappa} = 0\ , \tag{8}$$

where det $(\mathbf{z}+\mathbf{1}) \neq 0$, det $(\mathbf{y}+\mathbf{1}) \neq 0$.

Eq. (8) is called the "*symmetrically-mixed expression of Ohm's law*" or the "*normalized Ohm's law*".

And it follows that

(a) $\mathbf{p} = \mathbf{y}\ \mathbf{q}$ (or $\mathbf{p}\ \mathbf{q}^{-1} = \mathbf{y}$) (9)

(b) $\mathbf{q} = \mathbf{z}\ \mathbf{p}$ (or $\mathbf{q}\ \mathbf{p}^{-1} = \mathbf{z}$) (10)

(c) $\mathbf{p} + \mathbf{q} = \mathbf{1}$ (11)

Proof

(a) $(\mathbf{z}+\mathbf{1})\times$the left side$\times(\mathbf{y}+\mathbf{1})$

$$=(\mathbf{z}+\mathbf{1})\ (\mathbf{z}+\mathbf{1})^{-1}\ (\mathbf{y}+\mathbf{1})$$
$$= \mathbf{y}+\mathbf{1}$$

$(\mathbf{z}+\mathbf{1})\times$the right side$\times(\mathbf{y}+\mathbf{1})$

$$=(\mathbf{z}+\mathbf{1})\ \mathbf{y}\ (\mathbf{y}+\mathbf{1})^{-1}\ (\mathbf{y}+\mathbf{1})$$
$$= \mathbf{1}+\mathbf{y}$$

(b) $(\mathbf{y}+\mathbf{1})\times$the left side$\times(\mathbf{z}+\mathbf{1})$

$$=(\mathbf{y}+\mathbf{1})\ (\mathbf{y}+\mathbf{1})^{-1}\ (\mathbf{z}+\mathbf{1})$$
$$= \mathbf{z}+\mathbf{1}$$

$(\mathbf{y}+\mathbf{1})\times$the right side$\times(\mathbf{z}+\mathbf{1})$

$$=(\mathbf{y}+\mathbf{1})\ \mathbf{z}\ (\mathbf{z}+\mathbf{1})^{-1}\ (\mathbf{z}+\mathbf{1})$$
$$= \mathbf{1}+\mathbf{z}$$

(c) the left side$\times\mathbf{p}^{-1} = (\mathbf{p}+\mathbf{q})\mathbf{p}^{-1}=\mathbf{1}+\mathbf{q}\mathbf{p}^{-1}$

$$=\mathbf{1}+\mathbf{z}$$

the right side$\times\mathbf{p}^{-1}=\mathbf{p}^{-1}$

$$=\mathbf{z}+\mathbf{1}$$

5. Ordinal analysis of the network problem expressed by the new normalized Ohm's law

Fundamental equations of an electrical network are shown as

Current law : $\mathbf{D}\ \mathbf{i} = \mathbf{S}$

Voltage law : $\mathbf{R}\ \mathbf{u} = \mathbf{E}$

Normalized Ohm's law : $\mathbf{p}\ \mathbf{u} - \mathbf{q}\ \mathbf{i} = \mathbf{0}$

where[2) 3)]

$$\mathbf{S} = \mathbf{D}\ \mathbf{s},\quad \mathbf{i} - \mathbf{s} = \mathbf{I} = \mathbf{R}'\ \mathbf{J}$$

$$\mathbf{E} = \mathbf{R}\,\mathbf{e},\quad \mathbf{u} - \mathbf{e} = \mathbf{U} = \mathbf{D}'\,\mathbf{V}\;.$$

The point potential $\mathbf{V}$ and the loop current $\mathbf{J}$ are determined respectively by

$$\begin{aligned}\mathbf{V} &= (\mathbf{D}\mathbf{q}^{-1}\,\mathbf{p}\mathbf{D}')^{-1}\,(\mathbf{S}-\mathbf{D}\mathbf{q}^{-1}\,\mathbf{p}\mathbf{e}) \\ &= (\mathbf{D}\mathbf{q}^{-1}\,\mathbf{p}\mathbf{D}')^{-1}\,\mathbf{D}\,(\mathbf{s}-\mathbf{q}^{-1}\,\mathbf{p}\mathbf{e})\end{aligned} \tag{12}$$

$$\begin{aligned}\mathbf{J} &= (\mathbf{R}\mathbf{p}^{-1}\,\mathbf{q}\mathbf{R}')^{-1}\,(\mathbf{E}-\mathbf{R}\mathbf{p}^{-1}\,\mathbf{q}\mathbf{s}) \\ &= (\mathbf{R}\mathbf{p}^{-1}\,\mathbf{q}\mathbf{R}')^{-1}\,\mathbf{R}\,(\mathbf{e}-\mathbf{p}^{-1}\,\mathbf{q}\mathbf{s}).\end{aligned} \tag{13}$$

And the branch voltage drop $\mathbf{u}$ and the branch current $\mathbf{i}$ respectivcly are decided by

$$\mathbf{u}=\mathbf{D}'\,(\mathbf{D}\mathbf{q}^{-1}\,\mathbf{p}\mathbf{D}')^{-1}\,\mathbf{D}\,(\mathbf{s}-\mathbf{q}^{-1}\,\mathbf{p}\mathbf{e})+\mathbf{e} \tag{14}$$

$$\mathbf{i}=\mathbf{R}'\,(\mathbf{R}\mathbf{p}^{-1}\,\mathbf{q}\mathbf{R}')^{-1}\,\mathbf{R}\,(\mathbf{e}-\mathbf{p}^{-1}\,\mathbf{q}\mathbf{s})+\mathbf{s}\;. \tag{15}$$

6. Switching elements

Regarding the *switching element*, we newly use the parameter (p, q). Namely if it is "*short*",

$$\begin{cases} p = \dfrac{1}{1+z} = \dfrac{1}{1+0} = 1 \\ q = \dfrac{1}{1+y} = \dfrac{1}{1+\infty} = 0\,,\ \text{then}\ (p,\ q) = (1,\ 0), \end{cases} \tag{16}$$

and if it is "*open*",

$$\begin{cases} p = \dfrac{1}{1+z} = \dfrac{1}{1+\infty} = 0 \\ q = \dfrac{1}{1+y} = \dfrac{1}{1+0} = 1\,,\ \text{then}\ (p,\ q) = (0,\ 1). \end{cases} \tag{17}$$

And regarding the usual resistance, the values of resistance $z=r$, conductance $y=g$, p and q are shown in Table 1 and Fig. 2.

Tab. 1 r, g, $p = \dfrac{1}{r+1}$, $q = \dfrac{1}{g+1}$

r	0	1	2	3	4	5	6	7	8	9 …… ∞
g	∞	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$ …… 0
$p = \frac{1}{k+1}$	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$ …… 0
$q = \frac{1}{g+1}$	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{6}$	$\frac{6}{7}$	$\frac{7}{8}$	$\frac{8}{9}$	$\frac{9}{10}$ …… 1

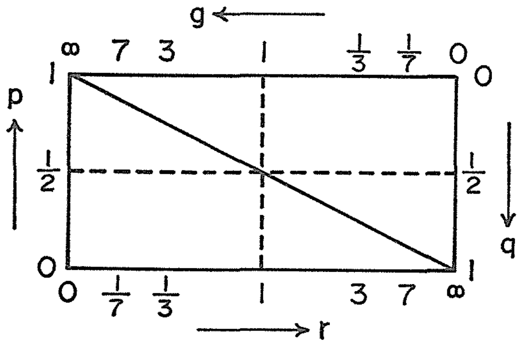

Fig. 2

7. Mixed analysis

In stead of fundamental equations

$$\begin{cases} \mathbf{D\,i} = \mathbf{s} \\ \mathbf{R\,u} = \mathbf{E} \\ \mathbf{p\,u} - \mathbf{p\,i} = 0 \end{cases}$$

using the set of equations

$$\begin{cases} \mathbf{I} = \mathbf{i} - \mathbf{s} = \mathbf{R'\,J} \quad \text{i. e.} \quad \mathbf{i} = \mathbf{R'\,J} + \mathbf{s} \\ \mathbf{U} = \mathbf{u} - \mathbf{e} = \mathbf{D'\,V} \quad \text{i. e.} \quad \mathbf{u} = \mathbf{D'\,V} + \mathbf{e} \\ \mathbf{p\,u} - \mathbf{q\,i} = 0 \end{cases}$$

we have

$$\mathbf{p\,D'\,V} - \mathbf{q\,R'\,J} = -\,\mathbf{p\,e} + \mathbf{q\,s}$$

$$p_{\mu\lambda}\, D^{b}_{\lambda}\, V_{b} - q_{\mu_\kappa}\, R^{\kappa}_{p}\, J^{p} = -p_{\mu\lambda}\, e_{\lambda} + q_{\mu_\kappa}\, s^{\kappa}\,. \tag{18}$$

Eq. (18) is called the "*mixed equation*", which has unknown quatities V_b and J^p and generally is useful especially for network problems with switching elements.

In conclusion, the mixed analysis of the electrical network with switching elements has been established in the symmetrical form such as aforementioned. And the several examples thereof are given as in the following section.

8. Examples

Ex. 1 (a) The mixed equation of Fig. 3 is shown as

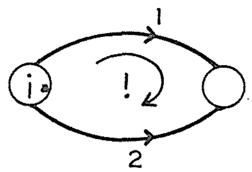

Fig. 3

$$
\begin{bmatrix} p_1{}^1 & p_1{}^2 \\ p_2{}^1 & p_2{}^2 \end{bmatrix}
\begin{bmatrix} 1 \\ 1 \end{bmatrix}
\begin{bmatrix} V_{\cdot i} \end{bmatrix}
-
\begin{bmatrix} p_{1\,1} & p_{1\,2} \\ p_{2\,1} & p_{2\,2} \end{bmatrix}
\begin{bmatrix} 1 \\ -1 \end{bmatrix}
\begin{bmatrix} J^{1}_{\cdot} \end{bmatrix}
$$

$$
= -
\begin{bmatrix} p_1{}^1 & p_1{}^2 \\ p_2{}^1 & p_2{}^2 \end{bmatrix}
\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}
+
\begin{bmatrix} p_{1\,1} & p_{1\,2} \\ p_{2\,1} & p_{2\,2} \end{bmatrix}
\begin{bmatrix} s^1 \\ s^2 \end{bmatrix}
\tag{19}
$$

(b)　If there is no mutual coupling, Eq. (19) is determined as follows:

$$
\begin{bmatrix} \overset{1}{p} & \cdot \\ \cdot & \overset{2}{p} \end{bmatrix}
\begin{bmatrix} 1 \\ 1 \end{bmatrix}
\begin{bmatrix} V_{\cdot i} \end{bmatrix}
-
\begin{bmatrix} q_1 & \cdot \\ \cdot & q_2 \end{bmatrix}
\begin{bmatrix} 1 \\ -1 \end{bmatrix}
\begin{bmatrix} J^{1}_{\cdot} \end{bmatrix}
$$

$$
= -
\begin{bmatrix} \overset{1}{p} & \cdot \\ \cdot & \overset{2}{p} \end{bmatrix}
\begin{bmatrix} e^1 \\ e^2 \end{bmatrix}
+
\begin{bmatrix} q_1 & \cdot \\ \cdot & q_2 \end{bmatrix}
\begin{bmatrix} s^1 \\ s^2 \end{bmatrix}
$$

$$
\therefore\quad
\left[\begin{array}{c|c} \overset{1}{p} & -q_1 \\ \overset{2}{p} & q_2 \end{array}\right]
\left[\begin{array}{c} V_{\cdot i} \\ \hline J^{1}_{\cdot} \end{array}\right]
= -
\begin{bmatrix} \overset{1}{p}\, e_1 \\ \overset{2}{p}\, e_2 \end{bmatrix}
+
\begin{bmatrix} q_1\, s^1 \\ q_2\, s^2 \end{bmatrix} .
\tag{20}
$$

(c)　When $e_2 = s^1 = s^2 = 0$, Eq. (20) is

$$
\left[\begin{array}{c|c} \overset{1}{p} & -q_1 \\ \overset{2}{p} & q_2 \end{array}\right]
\left[\begin{array}{c} V_{\cdot i} \\ \hline J^{1}_{\cdot} \end{array}\right]
= -
\begin{bmatrix} \overset{1}{p}\, e_1 \\ \cdot \end{bmatrix}
$$

$$
\therefore\quad
\left[\begin{array}{c} V_{\cdot i} \\ \hline J^{1}_{\cdot} \end{array}\right]
=
\left[\begin{array}{c|c} \overset{1}{p} & -q_1 \\ \overset{2}{p} & q_2 \end{array}\right]^{-1}
\begin{bmatrix} -\overset{1}{p}\, e_1 \\ \cdot \end{bmatrix}
=
\frac{
\begin{bmatrix} q_2 & q_1 \\ -\overset{2}{p} & \overset{1}{p} \end{bmatrix}
\begin{bmatrix} -\overset{1}{p}\, e_1 \\ \cdot \end{bmatrix}
}{\overset{1}{p}\, q_2 + \overset{2}{p}\, q_1}
$$

$$
= \frac{1}{\overset{1}{p}\, q_2 + \overset{2}{p}\, q_1}
\left[\begin{array}{c} -\overset{1}{p}\, q_2\, e_1 \\ \hline \overset{1}{p}\, \overset{2}{p}\, e_1 \end{array}\right] .
\tag{21}
$$

(d)　If $\underset{1}{r} = 1$,　$\underset{2}{r} = 2$,　then

$$(\overset{1}{p},\ \underset{1}{q}) = (\tfrac{1}{2},\ \tfrac{1}{2}),\ (\overset{2}{p},\ \underset{2}{q}) = (\tfrac{1}{3},\ \tfrac{2}{3}).$$

Therefore, Eq. (21) is shown as

$$\left\{\begin{aligned} V_{\dot{1}} &= \frac{-\frac{1}{2}\cdot\frac{2}{3}\,e_1}{\frac{1}{2}\cdot\frac{2}{3}+\frac{1}{3}\cdot\frac{1}{2}} = \frac{-\frac{2}{6}\,e_1}{\frac{2}{6}+\frac{1}{6}} = -\frac{2}{3}e_1 \\ J^{\underset{\cdot}{1}} &= \frac{\frac{1}{2}\cdot\frac{1}{3}\,e_1}{\frac{1}{2}\cdot\frac{2}{3}+\frac{1}{3}\cdot\frac{1}{2}} = \frac{\frac{1}{6}\,e_1}{\frac{2}{6}+\frac{1}{6}} = \frac{1}{3}e_1 \ . \end{aligned}\right. \tag{22}$$

(e) If there are switching element $(\overset{1}{p},\ \underset{1}{q})$ and resistance $\underset{2}{r}=2$ i. e.

$(\overset{2}{p},\ \underset{2}{q}) = (\frac{1}{3},\ \frac{2}{3})$, Eq. (21) is

$$\left\{\begin{aligned} V_{\dot{1}} &= \frac{-\overset{1}{p}\cdot\frac{2}{3}e_1}{\overset{1}{p}\cdot\frac{2}{3}+\frac{1}{3}\underset{1}{q}} = \frac{-2\overset{1}{p}}{2\overset{1}{p}+\underset{1}{q}}\,e_1 \\ J^{\underset{\cdot}{1}} &= \frac{\overset{1}{p}\cdot\frac{1}{3}e_1}{\overset{1}{p}\cdot\frac{2}{3}+\frac{1}{3}\underset{1}{q}} = \frac{\overset{1}{p}}{2\overset{1}{p}+\underset{1}{q}}\,e_1 \ . \end{aligned}\right. \tag{23}$$

(f) If element 1 is opened, $(\overset{1}{p},\ \underset{1}{q}) = (0,1)$, then Eq. (23) is

$$\left\{\begin{aligned} V_{\dot{1}} &= \frac{-2\cdot 0}{2\cdot 0+1}\,e_1 = 0 \\ J^{\underset{\cdot}{1}} &= \frac{0}{2\cdot 0+1}\,e_1 = 0 \ . \end{aligned}\right. \tag{24}$$

(g) If 1 is shortened, $(\overset{1}{p},\ \underset{1}{q}) = (1,\ 0)$, then Eq. (23) is

$$\left\{\begin{aligned} V_{\dot{1}} &= \frac{-2\cdot 1}{2\cdot 1+0}\,e_1 = -e_1 \\ J^{\underset{\cdot}{1}} &= \frac{1}{2\cdot 1+0} = \frac{1}{2}e_1 \ . \end{aligned}\right. \tag{25}$$

Ex. 2 (a) In Fig. 4, it is given that

$$D^{b}_{\lambda} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\!\!\begin{array}{c} \dot{1} \\ \left[\begin{array}{c} 1 \\ 1 \\ 1 \end{array}\right] \end{array}, \quad R^{\kappa}_{p} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\!\!\begin{array}{c} \underset{\cdot}{1}\ \ \underset{\cdot}{2} \\ \left[\begin{array}{cc} 1 & \cdot \\ -1 & 1 \\ \cdot & -1 \end{array}\right] \end{array}$$

$$p^{\mu\lambda} = \begin{bmatrix} \frac{1}{2} & \cdot & \cdot \\ \cdot & \frac{1}{3} & \cdot \\ \cdot & \cdot & \overset{3}{p} \end{bmatrix}, \qquad q^{\mu}{}_{\kappa} = \begin{bmatrix} \frac{1}{2} & \cdot & \cdot \\ \cdot & \frac{2}{3} & \cdot \\ \cdot & \cdot & \underset{3}{q} \end{bmatrix}$$

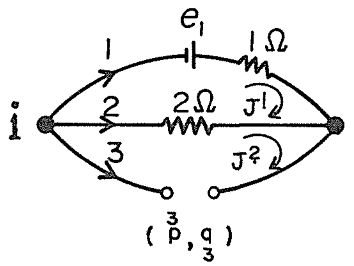

Fig. 4

and the mixed equation is shown as

$$\begin{bmatrix} \frac{1}{2} & \cdot & \cdot \\ \cdot & \frac{1}{3} & \cdot \\ \cdot & \cdot & \overset{3}{p} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} V_{i} \end{bmatrix} - \begin{bmatrix} \frac{1}{2} & \cdot & \cdot \\ \cdot & \frac{2}{3} & \cdot \\ \cdot & \cdot & \underset{3}{q} \end{bmatrix} \begin{bmatrix} 1 & \cdot \\ -1 & 1 \\ \cdot & -1 \end{bmatrix} \begin{bmatrix} J^{1}_{\cdot} \\ J^{2}_{\cdot} \end{bmatrix} = - \begin{bmatrix} \frac{1}{2} & \cdot & \cdot \\ \cdot & \frac{1}{3} & \cdot \\ \cdot & \cdot & \overset{3}{p} \end{bmatrix} \begin{bmatrix} e_1 \\ \cdot \\ \cdot \end{bmatrix}$$

$$\therefore \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \overset{3}{p} \end{bmatrix} \begin{bmatrix} V_{i} \end{bmatrix} - \begin{bmatrix} \frac{1}{2} & \cdot \\ -\frac{2}{3} & \frac{2}{3} \\ \cdot & -\underset{3}{q} \end{bmatrix} \begin{bmatrix} J^{1}_{\cdot} \\ J^{2}_{\cdot} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} e_1 \\ \cdot \\ \cdot \end{bmatrix}$$

$$\therefore \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \cdot \\ \frac{1}{3} & \frac{2}{3} & -\frac{2}{3} \\ \overset{3}{p} & \cdot & \underset{3}{q} \end{bmatrix} \begin{bmatrix} V_{i} \\ J^{1}_{\cdot} \\ J^{2}_{\cdot} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} e_1 \\ \cdot \\ \cdot \end{bmatrix}$$

$$\begin{bmatrix} V_{i} \\ J^{1}_{\cdot} \\ J^{2}_{\cdot} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \cdot \\ \frac{1}{3} & \frac{2}{3} & -\frac{2}{3} \\ \overset{3}{p} & \cdot & \underset{3}{q} \end{bmatrix}^{-1} \begin{bmatrix} -\frac{1}{2} e_1 \\ \cdot \\ \cdot \end{bmatrix}$$

$$
= \frac{\begin{array}{|ccc|c|} \frac{2}{3} \underset{3}{q} & \frac{1}{2} \underset{3}{q} & \frac{2}{6} & -\frac{1}{2} e_1 \\ -\left(\frac{2}{3} \overset{3}{p} + \frac{1}{3} \underset{3}{q} \right) & \frac{1}{2} \underset{3}{q} & \frac{1}{3} & \cdot \\ -\frac{2}{3} \overset{3}{p} & -\frac{1}{2} \overset{3}{p} & \frac{1}{2} & \cdot \end{array}}{\begin{vmatrix} \frac{1}{2} & -\frac{1}{2} & \cdot \\ \frac{1}{3} & \frac{2}{3} & -\frac{2}{3} \\ \overset{3}{p} & \cdot & \underset{3}{q} \end{vmatrix}}
$$

$$
= \frac{1}{\frac{1}{6} \left(2 \overset{3}{p} + 3 \underset{3}{q} \right)} \begin{bmatrix} -\frac{1}{3} \underset{3}{q} \, e_1 \\ \left(\frac{1}{3} \overset{3}{p} + \frac{1}{6} \underset{3}{q} \right) e_1 \\ \frac{1}{3} \overset{3}{p} \, e_1 \end{bmatrix}
$$

$$
= \frac{e_1}{2 \overset{3}{p} + 3 \underset{3}{q}} \begin{bmatrix} -2 \underset{3}{q} \\ 2 \overset{3}{p} + \underset{3}{q} \\ 2 \overset{3}{p} \end{bmatrix} . \tag{26}
$$

(b) If $(\overset{3}{p}, \underset{3}{q}) = (0, 1)$, Eq. (26) is

$$
\begin{cases} V_{\dot{1}} = -\frac{2}{3} e_1 \\ J^{1}_{\cdot} = \frac{1}{3} e_1 \\ J^{2}_{\cdot} = 0 . \end{cases} \tag{27}
$$

(c) If $(\overset{3}{p}, \underset{3}{q}) = (1, 0)$, Eq. (26) is

$$
\begin{cases} V_{\dot{1}} = 0 \\ J^{1}_{\cdot} = e_1 \\ J^{2}_{\cdot} = e_1 . \end{cases}
$$

Ex. 3 (a) In Fig. 5, when there is only e_1, the mixed equation is given by

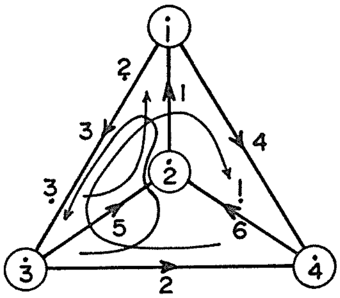

Fig. 5

$$\left[\begin{array}{ccc|ccc} -\overset{1}{p} & \overset{1}{p} & \cdot & \cdot & q_1 & q_1 \\ \cdot & \cdot & \overset{2}{p} & -q_2 & \cdot & q_2 \\ \overset{3}{p} & \cdot & -\overset{3}{p} & -q_3 & q_3 & q_3 \\ \overset{4}{p} & \cdot & \cdot & q_4 & \cdot & \cdot \\ \cdot & -\overset{5}{p} & \overset{5}{p} & \cdot & q_5 & \cdot \\ \cdot & -\overset{6}{p} & \cdot & \cdot & \cdot & q_6 \end{array}\right] \left[\begin{array}{c} V_{\dot{1}} \\ V_{\dot{2}} \\ V_{\dot{3}} \\ \hline -J^{\dot{1}} \\ -J^{\dot{2}} \\ -J^{\dot{3}} \end{array}\right] = \left[\begin{array}{c} -\overset{1}{p} e_1 \\ \cdot \\ \cdot \\ \hline \cdot \\ \cdot \\ \cdot \end{array}\right] . \qquad (28)$$

(b) If $r_1,\ r_2,\ r_3,\ r_4,\ r_5 = 1\,\Omega$, then

$$\overset{1}{p},\ \overset{2}{p},\ \overset{3}{p},\ \overset{4}{p},\ \overset{5}{p} = \frac{1}{2} \text{ and } q_1,\ q_2,\ q_3,\ q_4,\ q_5 = \frac{1}{2} .$$

Thercfore, Eq. (28) is

$$\left[\begin{array}{ccc|ccc} -\frac{1}{2} & \frac{1}{2} & \cdot & \cdot & \frac{1}{2} & \frac{1}{2} \\ \cdot & \cdot & \frac{1}{2} & -\frac{1}{2} & \cdot & \frac{1}{2} \\ \frac{1}{2} & \cdot & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \cdot & \cdot & \frac{1}{2} & \cdot & \cdot \\ \cdot & -\frac{1}{2} & \frac{1}{2} & \cdot & \frac{1}{2} & \cdot \\ \cdot & -\overset{6}{p} & \cdot & \cdot & \cdot & q_6 \end{array}\right] \left[\begin{array}{c} V_{\dot{1}} \\ V_{\dot{2}} \\ V_{\dot{3}} \\ \hline -J^{\dot{1}} \\ -J^{\dot{2}} \\ -J^{\dot{3}} \end{array}\right] = \left[\begin{array}{c} -\frac{1}{2} e_1 \\ \cdot \\ \cdot \\ \hline \cdot \\ \cdot \\ \cdot \end{array}\right] .$$

One of the solutions $V_{\dot{1}}$ is given by

$$V_1 = \frac{2^{-5}\begin{vmatrix} -1 & 1 & \cdot & \cdot & 1 & 1 \\ \cdot & \cdot & 1 & -1 & \cdot & 1 \\ \cdot & \cdot & -1 & -1 & 1 & 1 \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & -1 & 1 & \cdot & 1 & \cdot \\ \cdot & -\overset{6}{p} & \cdot & \cdot & \cdot & q_6 \end{vmatrix}}{2^{-5}\begin{vmatrix} -1 & 1 & \cdot & \cdot & 1 & 1 \\ \cdot & \cdot & 1 & -1 & \cdot & 1 \\ 1 & \cdot & -1 & -1 & 1 & 1 \\ 1 & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & -1 & 1 & \cdot & 1 & \cdot \\ \cdot & -\overset{6}{p} & \cdot & \cdot & \cdot & q_6 \end{vmatrix}} e_1 = \frac{3\overset{6}{p}+q_6}{8\overset{6}{p}+8q_6} e_1 . \tag{29}$$

(c) If the above network involves resistances $r_1, r_4, r_5, r_6 = 1\Omega$ and switching elements $(\overset{2}{p}, q_2)$, $(\overset{3}{p}, q_3)$, it holds that

$$\overset{1}{p}, \overset{4}{p}, \overset{5}{p}, \overset{6}{p} = \frac{1}{2}, \quad q_1, q_4, q_5, q_6 = \frac{1}{2}$$

and the value of the point potential V_1 is given by

$$V_1 = \frac{2^{-4}\begin{vmatrix} -1 & 1 & \cdot & \cdot & 1 & 1 \\ \cdot & \cdot & \overset{2}{p} & -q_2 & \cdot & q_2 \\ \cdot & \cdot & -\overset{3}{p} & -q_3 & q_3 & q_3 \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & -1 & 1 & \cdot & 1 & \cdot \\ \cdot & -1 & \cdot & \cdot & \cdot & 1 \end{vmatrix}}{2^{-4}\begin{vmatrix} -1 & 1 & \cdot & \cdot & 1 & 1 \\ \cdot & \cdot & \overset{2}{p} & -q_2 & \cdot & q_2 \\ \overset{3}{p} & \cdot & -\overset{3}{p} & -q_3 & q_3 & q_3 \\ 1 & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & -1 & 1 & \cdot & 1 & \cdot \\ \cdot & -1 & \cdot & \cdot & \cdot & 1 \end{vmatrix}} e_1 = \frac{2\overset{2}{p}q_3+\overset{3}{p}q_2+q_2q_3}{3\overset{2}{p}\overset{3}{p}+5\overset{2}{p}q_3+5\overset{3}{p}q_2+3q_2q_3} e_1 . \tag{30}$$

Ex. 4 In Fig. 6, the resistance network with *time-variable switching elements*, namely periodically-interrupted ones S_1 and S_2 is shown. If the action of S_1 and S_2 and the value of e.m.f. e_1 are shown in Fig. 7, these values can be written as

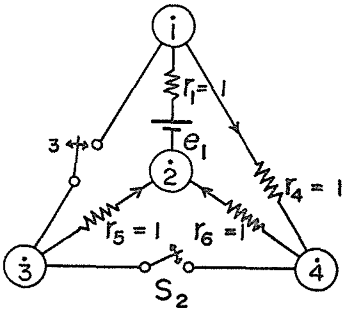

Fig. 6

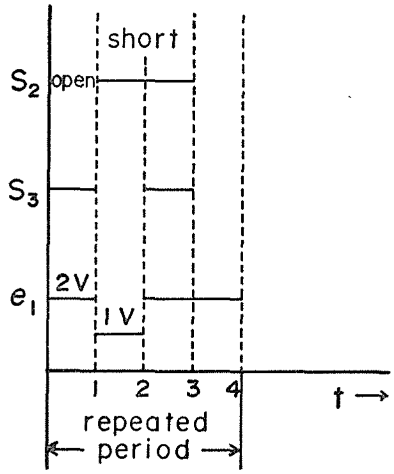

Fig. 7

$$e_1 = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 2 \end{bmatrix}, \quad S_2 = (\overset{2}{p}, \underset{2}{q}) = \left(\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right),$$

$$S_3 = (\overset{3}{p}, \underset{3}{q}) = \left(\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right).$$

By Eq. (30), we have

$$V_{\dot{i}} = \frac{2 \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}}{3 \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + 5 \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} + 5 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + 3 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}} \begin{bmatrix} 2 \\ 1 \\ 2 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2/5 \\ 2/5 \\ 0 \\ 2/3 \end{bmatrix}.$$

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開閉素子をもつ電気回路網の対称混合解析

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先に著者の中の二人は電気回路網の基礎方程式の混合表現と図解法を発表し，更にその一人と協力者は開閉素子を含む回路への拡張に成功した。今回はその基礎方程式の点電位・閉路電流による対称混合表現を求め，その解析例を紹介する。また，その実用例として，時変開閉素子をもつ抵抗回路の解析をあげた。